Present Value, Future Value

Future Value = Present Value + Interest Amount
Interest Amount = Principal Amount x Interest Rate

Future Value of a Single Present Amount

Future Value = Present Amount x \((1 + r)^n\)
Future Value = Present Amount x Future Value (FV) factor for a single present amount
FV factor for a single present amount = \((1 + r)^n\)
\(r\) = interest rate or discount rate
\(n\) = number of periods

<table>
<thead>
<tr>
<th>PV at t</th>
<th>FV at t+1</th>
<th>FV at t+2</th>
<th>FV at t+3</th>
<th>.......</th>
<th>FV at t+n</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1 + r))</td>
<td>((1 + r))</td>
<td>((1 + r))</td>
<td>((1 + r))</td>
<td></td>
<td>((1 + r))</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
FV \text{ at } t+1 &= PV \times (1 + r) \\
FV \text{ at } t+2 &= PV \times (1 + r) \times (1 + r) = PV \times (1 + r)^2 \\
FV \text{ at } t+3 &= PV \times (1 + r) \times (1 + r) \times (1 + r) = PV \times (1 + r)^3 \\
FV \text{ at } t+n &= PV \times (1 + r)^n \\
\end{align*}
\]

Present Value of a Single Future Amount

Present Value = Future Amount x \(\frac{1}{(1 + r)^n}\)
Present Value = Future Amount x Present Value (PV) factor for a single future amount
PV factor for a single future amount = \(\frac{1}{(1 + r)^n}\)
\(r\) = interest rate or discount rate
\(n\) = number of periods

<table>
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<th>PV at t</th>
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<td></td>
<td>((1 + r))</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
PV \text{ at } t &= FV \text{ at } t+1 \times \frac{1}{(1 + r)} \\
PV \text{ at } t &= FV \text{ at } t+2 \times \frac{1}{(1 + r)^2} \\
PV \text{ at } t &= FV \text{ at } t+3 \times \frac{1}{(1 + r)^3} \\
PV \text{ at } t &= FV \text{ at } t+n \times \frac{1}{(1 + r)^n} \\
\end{align*}
\]
Future Value of an Ordinary Annuity

Future Value = Annuity Amount x \( \frac{(1+r)^n - 1}{r} \)

Future Value = Annuity Amount x Future Value (FV) factor for an ordinary annuity

FV factor for an ordinary annuity = \( \frac{(1+r)^n - 1}{r} \)

\( r \) = interest rate or discount rate
\( n \) = number of periods

<table>
<thead>
<tr>
<th></th>
<th>( (1+r) )</th>
<th>( (1+r) )</th>
<th>( (1+r) )</th>
<th>( (1+r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV at t</td>
<td>Annuity at t+1</td>
<td>Annuity at t+2</td>
<td>Annuity at t+3</td>
<td>( \ldots )</td>
</tr>
</tbody>
</table>

Annuity at t+1 = Annuity at t+2 = Annuity at t+3 = \( \ldots \) = Annuity at t+n

Ordinary Annuity: Same amount is paid at the end of each period.

Future Value of an Ordinary Annuity

\[
= \text{Annuity} + \text{Annuity} \times (1+r) + \text{Annuity} \times (1+r)^2 + \text{Annuity} \times (1+r)^3 + \ldots + \text{Annuity} \times (1+r)^{n-1}
\]

\[
= \text{Annuity} \times [1 + (1+r) + (1+r)^2 + (1+r)^3 + \ldots + (1+r)^{n-1}]
\]

\[
= \text{Annuity} \times \frac{(1+r)^n - 1}{r}
\]

Geometric Series: \( 1 + k + k^2 + k^3 + \ldots + k^{n-1} = \frac{1-k^n}{1-k} \)

Present Value of an Ordinary Annuity

Present Value = Annuity Amount x \( \left( \frac{1 - \frac{1}{(1+r)^n}}{r} \right) \)

Present Value = Annuity Amount x Present Value (PV) factor for an ordinary annuity

PV factor for an ordinary annuity = \( \left( \frac{1 - \frac{1}{(1+r)^n}}{r} \right) \)

\( r \) = interest rate or discount rate
\( n \) = number of periods
Present Value of an Ordinary Annuity

\[
\text{PV at } t = \text{FV at } t+1 \times \frac{1}{(1 + r)}
\]

Annuity at \(t+1\) = Annuity at \(t+2\) = Annuity at \(t+3\) = \ldots = Annuity at \(t+n\)

Ordinary Annuity: Same amount is paid at the end of each period.

Ordinary Annuity vs. Annuity Due

Ordinary annuity: Same amount is paid at the end of each period.

Annuity due: Same amount is paid at the beginning of each period.