Present Value, Future Value

Future Value = Present Value + Interest Amount
Interest Amount = Principal Amount x Interest Rate

Future Value of a Single Present Amount

Future Value = Present Amount x $(1 + r)^n$
Future Value = Present Amount x Future Value (FV) factor for a single present amount
FV factor for a single present amount = $(1 + r)^n$
r = interest rate or discount rate
n = number of periods

	(1+ r)	(1+ r)	(1+ r)		(1+ r)
PV at t	FV at t+1	FV at t+2	FV at t+3	…….	FV at t+n

FV at t+1 = PV x $(1 + r)$
FV at t+2 = PV x $(1 + r)$ x $(1 + r)$ = PV x $(1 + r)^2$
FV at t+3 = PV x $(1 + r)$ x $(1 + r)$ x $(1 + r)$ = PV x $(1 + r)^3$
FV at t+n = PV x $(1 + r)^n$

Present Value of a Single Future Amount

Present Value = Future Amount x $\frac{1}{(1+r)^n}$
Present Value = Future Amount x Present Value (PV) factor for a single future amount
PV factor for a single future amount = $\frac{1}{(1+r)^n}$
r = interest rate or discount rate
n = number of periods

	(1+ r)	(1+ r)	(1+ r)		(1+ r)
PV at t	FV at t+1	FV at t+2	FV at t+3	…….	FV at t+n

PV at t = FV at t+1 x $\frac{1}{(1 + r)}$

PV at t = FV at t+2 x $\frac{1}{(1 + r)^2}$

PV at t = FV at t+3 x $\frac{1}{(1 + r)^3}$

PV at t = FV at t+n x $\frac{1}{(1 + r)^n}$

Future Value of an Ordinary Annuity

Future Value = Annuity Amount x $\frac{(1+r)^n - 1}{r}$

Future Value = Annuity Amount x Future Value (FV) factor for an ordinary annuity

FV factor for an ordinary annuity = $\frac{(1+r)^n - 1}{r}$

r = interest rate or discount rate
n = number of periods

	(1+ r)	(1+ r)	(1+ r)		(1+ r)
PV at t	Annuity at t+1	Annuity at t+2	Annuity at t+3	…….	Annuity at t+n

Annuity at t+1 = Annuity at t+2 = Annuity at t+3 = ……. = Annuity at t+n

Ordinary Annuity: Same amount is paid at the end of each period.

Future Value of an Ordinary Annuity
= Annuity + Annuity x (1 + r) + Annuity x $(1 + r)^2$ + Annuity x $(1 + r)^3$ +……+ Annuity x $(1 + r)^{n-1}$

= Annuity x [1 + (1 + r) + $(1 + r)^2$ + $(1 + r)^3$ +……+ $(1 + r)^{n-1}$]

= Annuity x $\frac{(1+r)^n - 1}{r}$

Geometric Series: $1 + k + k^2 + k^3 + ... + k^{n-1} = \frac{(1-k^n)}{1-k}$

Present Value of an Ordinary Annuity

Present Value = Annuity Amount x $\left(\frac{1 - \frac{1}{(1+r)^n}}{r} \right)$

Present Value = Annuity Amount x Present Value (PV) factor for an ordinary annuity

PV factor for an ordinary annuity = $\left(\frac{1 - \frac{1}{(1+r)^n}}{r} \right)$

r = interest rate or discount rate
n = number of periods

	(1+ r)	(1+ r)	(1+ r)		(1+ r)
PV at t	Annuity at t+1	Annuity at t+2	Annuity at t+3	…….	Annuity at t+n

$$\text{PV at t} = \text{FV at t+1} \times \frac{1}{(1+r)}$$

Annuity at t+1 = Annuity at t+2 = Annuity at t+3 = ……. = Annuity at t+n
Ordinary Annuity: Same amount is paid at the end of each period.

Present Value of an Ordinary Annuity

$$= \text{Annuity} \times \frac{1}{(1+r)} + \text{Annuity} \times \frac{1}{(1+r)^2} + \text{Annuity} \times \frac{1}{(1+r)^3} + \ldots\ldots + \text{Annuity} \times \frac{1}{(1+r)^n}$$

$$= \text{Annuity} \times \left(\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \ldots\ldots + \frac{1}{(1+r)^n}\right)$$

$$= \text{Annuity} \times \left(\frac{1-\frac{1}{(1+r)^n}}{r}\right)$$

$$\text{As } n \rightarrow \infty, \quad \left(\frac{1-\frac{1}{(1+r)^n}}{r}\right) \rightarrow \frac{1}{r}$$

Ordinary Annuity vs. Annuity Due

Ordinary annuity: Same amount is paid at the end of each period.
Annuity due: Same amount is paid at the beginning of each period.