

Present Value, Future Value

Future Value = Present Value + Interest Amount
 Interest Amount = Principal Amount x Interest Rate

Future Value of a Single Present Amount

Future Value = Present Amount x $(1 + r)^n$
 Future Value = Present Amount x Future Value (FV) factor for a single present amount
 FV factor for a single present amount = $(1 + r)^n$
 r = interest rate or discount rate
 n = number of periods

	(1 + r)	(1 + r)	(1 + r)		(1 + r)
PV at t	FV at t+1	FV at t+2	FV at t+3	FV at t+n

$$\begin{aligned} \text{FV at } t+1 &= \text{PV} \times (1 + r) \\ \text{FV at } t+2 &= \text{PV} \times (1 + r) \times (1 + r) = \text{PV} \times (1 + r)^2 \\ \text{FV at } t+3 &= \text{PV} \times (1 + r) \times (1 + r) \times (1 + r) = \text{PV} \times (1 + r)^3 \\ \text{FV at } t+n &= \text{PV} \times (1 + r)^n \end{aligned}$$

Present Value of a Single Future Amount

Present Value = Future Amount x $\frac{1}{(1 + r)^n}$
 Present Value = Future Amount x Present Value (PV) factor for a single future amount
 PV factor for a single future amount = $\frac{1}{(1 + r)^n}$
 r = interest rate or discount rate
 n = number of periods

	(1 + r)	(1 + r)	(1 + r)		(1 + r)
PV at t	FV at t+1	FV at t+2	FV at t+3	FV at t+n

$$\begin{aligned} \text{PV at } t &= \text{FV at } t+1 \times \frac{1}{(1 + r)} \\ \text{PV at } t &= \text{FV at } t+2 \times \frac{1}{(1 + r)^2} \\ \text{PV at } t &= \text{FV at } t+3 \times \frac{1}{(1 + r)^3} \\ \text{PV at } t &= \text{FV at } t+n \times \frac{1}{(1 + r)^n} \end{aligned}$$

Future Value of an Ordinary Annuity

$$\text{Future Value} = \text{Annuity Amount} \times \frac{(1+r)^n - 1}{r}$$

Future Value = Annuity Amount x Future Value (FV) factor for an ordinary annuity

$$\text{FV factor for an ordinary annuity} = \frac{(1+r)^n - 1}{r}$$

r = interest rate or discount rate

n = number of periods

	(1+r)	(1+r)	(1+r)		(1+r)
PV at t	Annuity at t+1	Annuity at t+2	Annuity at t+3	Annuity at t+n

Annuity at t+1 = Annuity at t+2 = Annuity at t+3 = = Annuity at t+n

Ordinary Annuity: Same amount is paid at the end of each period.

Future Value of an Ordinary Annuity

$$= \text{Annuity} + \text{Annuity} \times (1+r) + \text{Annuity} \times (1+r)^2 + \text{Annuity} \times (1+r)^3 + \dots + \text{Annuity} \times (1+r)^{n-1}$$

$$= \text{Annuity} \times [1 + (1+r) + (1+r)^2 + (1+r)^3 + \dots + (1+r)^{n-1}]$$

$$= \text{Annuity} \times \frac{(1+r)^n - 1}{r}$$

$$\text{Geometric Series: } 1 + k + k^2 + k^3 + \dots + k^{n-1} = \frac{(1-k^n)}{1-k}$$

Present Value of an Ordinary Annuity

$$\text{Present Value} = \text{Annuity Amount} \times \left(\frac{1 - \frac{1}{(1+r)^n}}{r} \right)$$

Present Value = Annuity Amount x Present Value (PV) factor for an ordinary annuity

$$\text{PV factor for an ordinary annuity} = \left(\frac{1 - \frac{1}{(1+r)^n}}{r} \right)$$

r = interest rate or discount rate

n = number of periods

	(1 + r)	(1 + r)	(1 + r)		(1 + r)
PV at t	Annuity at t+1	Annuity at t+2	Annuity at t+3	Annuity at t+n

$$PV \text{ at } t = FV \text{ at } t+1 \times \frac{1}{(1+r)}$$

Annuity at t+1 = Annuity at t+2 = Annuity at t+3 = = Annuity at t+n
 Ordinary Annuity: Same amount is paid at the end of each period.

Present Value of an Ordinary Annuity

$$\begin{aligned} &= \text{Annuity} \times \frac{1}{(1+r)} + \text{Annuity} \times \frac{1}{(1+r)^2} + \text{Annuity} \times \frac{1}{(1+r)^3} + \dots + \text{Annuity} \times \frac{1}{(1+r)^n} \\ &= \text{Annuity} \times \left(\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \dots + \frac{1}{(1+r)^n} \right) \\ &= \text{Annuity} \times \left(\frac{1 - \frac{1}{(1+r)^n}}{r} \right) \end{aligned}$$

$$\text{As } n \rightarrow \infty, \left(\frac{1 - \frac{1}{(1+r)^n}}{r} \right) \rightarrow \frac{1}{r}$$

Ordinary Annuity vs. Annuity Due

Ordinary annuity: Same amount is paid at the end of each period.
 Annuity due: Same amount is paid at the beginning of each period.